ASSIGNMENT 4

DUE: Wednesday October 9, 5 PM. DO NOT COPY. ACKNOWLEDGE YOUR SOURCES.

Please read http://www.student.cs.uwaterloo.ca/~cs341 for general instructions and policies. When you are asked to give an algorithm you should: (1) describe the idea of your algorithm clearly in English; (2) give pseudocode; (3) argue correctness; (4) and analyze the run time.

1. [10 marks] **Photo display.** You are in charge of a photo contest, and must display all the photos sent in to the contest. You have photos $p_1, p_2, \ldots, p_n$, where photo $p_i$ has width $w_i$ and height $h_i$ (these need not be standard dimensions). The photos must remain in order. You need to place them in rows of maximum width $W$. Photos $p_i, \ldots, p_j$ can go in a row $r$ only if $\sum_{k=i}^{j} w_k \leq W$; the height of this row $r$ is defined to be $\max\{h_k : i \leq k \leq j\}$. The goal is to pack the photos into rows to minimize the sum of the heights of the rows.

For example, 10 photos\(^\text{1}\) can be arranged into three rows of maximum width $W$ in the following way:

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W
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In this arrangement, the photos 1–3 are in the first row, photos 4–6 are in the second row, and photos 7–10 are in the third row. The total height of the three rows is $h_1 + h_5 + h_9$.

\(^1\)Photos are from the Wright collection in the Library of congress.
(a) [2 marks] Consider the greedy algorithm that starts with $p_1$ and fills each row with as many photos as possible so long as the sum of widths is $\leq W$. Show that this algorithm does not always find an optimum solution.

(b) [8 marks] Give a dynamic programming algorithm to find the minimum sum of row heights.

*Hint.* Use $n$ subproblems. A runtime of $O(n^2)$ is possible, but you can get almost full marks for a clear solution with an $O(n^3)$ runtime.

2. [10 marks] **Maximizing the sum.** Suppose you are given a sequence of integers separated by + and – signs; for example:

$$1 + 3 - 2 - 5 + 1 - 6 + 7$$

You can change the value of this expression by adding parentheses in different places. For example:

$$1 + 3 - 2 - 5 + 1 - 6 + 7 = -1$$

$$(1 + 3 - (2 - 5)) + (1 - 6) + 7 = 9$$

$$1 + (3 - 2) - (5 + 1) - (6 + 7) = -17$$

$$1 + 3 - (2 - 5 + 1 - 6) + 7 = 19.$$ 

Give an algorithm to find the maximum value that can be attained by adding parentheses to a given sequence of the form $x_1, s_1, x_2, s_1, \ldots, s_{n-1}, x_n$ where each $x_i$ is an integer and each $s_i$ is an arithmetic operator + or –. Note that you may not use parentheses to create implicit multiplications as in $1 + 3(-2)(-5) + 1 - 6 + 7 = 33$.

*Hint.* You should aim to solve two subproblems for each pair of indices $i, j$, $1 \leq i \leq j \leq n$. 