DUE: Wednesday October 4, 7 PM. DO NOT COPY. ACKNOWLEDGE YOUR SOURCES.

Please read http://www.student.cs.uwaterloo.ca/~cs341 for general instructions and policies.

1. [20 marks] **Divide-and-conquer.** The input for this problem consists of \( n \) radar stations where station \( i \) is given by its integer coordinates \( x_i \) and \( y_i \) in the plane. We say that station \( i \) can transmit to station \( j \) if station \( j \) is south-west of station \( i \), i.e., \( x_i \geq x_j \) and \( y_i \geq y_j \). The “load factor” of station \( i \) is defined to be the number of stations it can transmit to (not counting itself). The goal is to compute the load factor of each station. You will develop and implement two algorithms for this problem.

**Input and output.** The input consists of \( n + 1 \) lines. The first line has \( n \), the number of stations. The following \( n \) lines have three numbers each: the ID number of station \( i \), and its \( x_i \) and \( y_i \) coordinates. All station IDs are different. We will not be testing whether your program detects input errors.

The output must have \( n \) lines, each giving a station ID number and the load factor for that station. Your output must be sorted by station ID (in increasing order).

**Sample input**

\[
\begin{array}{c}
4 \\
2 3 3 \\
10 4 2 \\
13 5 6 \\
7 2 1
\end{array}
\]

**Sample output**

\[
\begin{array}{c}
2 1 \\
7 0 \\
10 1 \\
13 3
\end{array}
\]

(a) Implement a brute-force algorithm for this problem. For each radar station \( i \), go through the whole list of radar stations to find the ones that \( i \) can transmit to. Your algorithm will have worst-case run time \( \Theta(n^2) \).

(b) Implement an efficient divide-and-conquer algorithm for this problem. Your algorithm should have worst-case run time \( \Theta(n \log n) \). (Our tests will check that your algorithm is fast enough.) Hints for the algorithm are as follows:
Sort the stations by $x_i$ and by $y_i$, creating arrays $X$ and $Y$.

Write a recursive routine that takes as input a set of radar stations given as two arrays, array $X$ sorted by $x_i$ and array $Y$ sorted by $y_i$:

- Divide the stations into Left half and Right half based on $x_i$
- Construct arrays $X_1$, $X_2$ of stations on the Left/Right sorted by $x_i$.
- Construct arrays $Y_1$, $Y_2$ of stations on the Left/Right sorted by $y_i$. Be careful to take linear time.
- Recurse on $X_1$, $Y_1$ and on $X_2$, $Y_2$.
- Combine the solutions to the two halves. Be careful to take linear time.
  - Observe that if station $i$ is in the Left half then you have the correct load factor.
  - If a station $i$ is in the Right half, then $i$ can also transmit to all stations $j$ in the Left half with $y_j \leq y_i$. You can find the number of such stations by using array $Y_1$. 
