CS 360 — Introduction to the Theory of Computing
Assignment 2

University of Waterloo, Spring 2018

Due 5:00 PM, June 1, 2018.

In this assignment, you are asked to determine whether or not the given language \( L \) is regular. To show that \( L \) is regular, provide a finite automaton or regular expression and/or argue using closure properties of regular languages. You do not need to make a formal induction argument—an informal argument that is enough to convince the marker will suffice. To show that \( L \) is not regular, provide an argument using the pumping lemma and/or closure properties of regular languages.

1. Let \( L = \{a^k b w \mid w \in \{a,b\}^k, k \in \mathbb{N}\} \). Show whether or not \( L \) is regular.

2. Let \( L = \{a^i b^j \mid i + j = 0 \mod 4; i, j \in \mathbb{N}\} \). Show whether or not \( L \) is regular.

3. Let \( L = \{a^m \mid m \neq n^2 \text{ for any } n \in \mathbb{N}\} \). Show whether or not \( L \) is regular.

4. Let \( \Sigma = \{A, C, G, T\} \) be the alphabet of DNA bases. Let \( \theta : \Sigma^* \rightarrow \Sigma^* \) be an antimorphism. An antimorphism is a mapping such that for \( u, v \in \Sigma^* \), we have \( \theta(uv) = \theta(v)\theta(u) \). We define \( \theta \) by

\[
\begin{align*}
\theta(A) &= T \\
\theta(C) &= G \\
\theta(G) &= C \\
\theta(T) &= A.
\end{align*}
\]

We call \( \theta \) the Watson-Crick involution; given a DNA strand \( w \in \Sigma^* \), \( \theta(w) \) is the Watson-Crick complement of \( w \). Observe that \( \theta \) is an involution, which means that it has the property that for any word \( w \in \Sigma^* \), we have \( \theta^2(w) = \theta(\theta(w)) = w \).

Let \( L = \{w \in \Sigma^* \mid w = \theta(w)\} \). Show whether or not \( L \) is regular.

5. Let \( G \) be the following context-free grammar.

\[
\begin{align*}
S &\rightarrow aSc \mid A \\
A &\rightarrow bA \mid \varepsilon
\end{align*}
\]

(a) What is the language generated by \( G \)? Argue informally about how \( G \) generates this language.

(b) Show whether or not \( L(G) \) is regular.