1. [10 marks] Prove by induction that $\epsilon^n = \epsilon$, where $\epsilon$ denotes the empty string.

2. [10 marks]
Recall the recursive definition of power of a string given in the slides: $x^0 = \epsilon$, and $x^i = x \cdot x^{i-1}$ for $i \geq 1$.
Using this definition, prove by induction that $x^{m+n} = x^m x^n$ for all strings $x$ and all integers $m, n \geq 0$. Think about what to induct on, and state it explicitly!

3. [10 marks]
Recall the recursive definition of reversal of a string given in the slides: $\epsilon^R = \epsilon$, and $(xa)^R = a \cdot x^R$ for strings $x$ and single symbol $a$.
Prove by induction that $(x^R)^n = (x^n)^R$ for all strings $x$ and all integers $n \geq 0$.
Again, think about what to induct on, and state it explicitly!