University of Waterloo
CS 360 — Introduction to the Theory of Computing
Fall 2017
Problem Set 1

Handed out Thursday, September 7, 2017

Due Thursday, September 14, 2017, at 5 PM. Submit to LEARN.

Finite automata can be described in at least two ways: by drawing the transition diagram, or by explicitly describing all five pieces $Q, \Sigma, \delta, q_0, F$ that define the automaton. In this problem set, you will do both.

First, transition diagrams:

1. [20 marks] For each of the following, draw the transition diagram of a deterministic finite automaton (DFA) that recognizes the specified language. Make sure that the initial state is clearly labeled, and that all accepting states are drawn with two concentric circles. Be sure to draw a *complete* automaton, so that every state has an arrow out for each letter in the alphabet. Briefly explain your construction in English.

Try to find the smallest DFA (that is, the smallest number of states) you can. Although it is not necessary to find the smallest possible automaton, marks will be deducted if your automaton is excessively complicated. The most crucial part of your construction, however, is your English explanation. No formal proof is necessary.

(a) [5 marks] The set of strings over $\{a, b\}$ containing at most one pair of two consecutive a's and at most one pair of two consecutive b's.

Note: the string aaa is considered to contain *two* pairs of two consecutive a's, which overlap each other.

(b) [5 marks] The set of nonempty strings x over $\{a, b\}$ such that every other symbol (starting with the *last* symbol and working backwards) is an a. For example, a, aa, ba, and baaaba are included in the specification, but ϵ and abbaa are not.

(c) [5 marks] The set of strings over $\{a, b, c\}$ such that at least one of the three symbols appears at least twice (not necessarily consecutively).

(d) [5 marks] The set of strings over $\{a, b\}$ that contain abaab as a subword.

Next, we'll turn to the formal definition. "Describe" means "define all five pieces $Q, \Sigma, \delta, q_0, F$".

2. [10 marks]

(a) [5 marks] For integers $n \geq 1$, describe a DFA recognizing the language

$$L_n = \{x \in \{0, 1\}^* : x \text{ has a 1 in the position } n \text{ places from the end}\}.$$

(Here the very last symbol is 1 place from the end.)

(b) [5 marks] For integers $n \geq 1$, describe a DFA recognizing the finite language

$$\{a^i b^i \ : 0 \leq i \leq n\}.$$