1. [10 marks] Call a string an *avatar* if it is nonempty and every odd-indexed letter is the same as the first. (The first letter has index 1, the second has index 2, etc.) So each member of \{gaga, gage, eleve, eleven\} is an avatar, but bananas is not an avatar.

Let \(L_{av}\) be the language of all avatars over the alphabet \{a, b\}.

Draw the transition diagram of a DFA recognizing the language \(L_{av}\). Be sure your DFA is *complete*: it has transitions on every state and every letter.

Explain how your automaton works. One way to do this is to explain the meaning of each state.

Try to make your DFA have as few states as possible. Marks will be deducted if it is excessively complicated.

2. [10 marks] Find regular expressions for the following languages. You don’t need to justify your answers, and you don’t need to find the shortest possible regular expression, but needlessly complicated expressions will have some amount of marks deducted.

   (a) [5 marks] Nonempty strings over the alphabet \{a, b\} whose first and last letter are the same.

   (b) [5 marks] The language \(L_{av}\) from problem 1.

If you find shorter expressions than the ones in the solutions I post, you will get 1 bonus mark for each shorter expression.

3. [10 marks] Recall the extended transition function \(\delta^*\) for DFA’s, defined recursively by \(\delta^*(q, \epsilon) = q\) and \(\delta^*(q, xa) = \delta(\delta^*(q, x), a)\) for all strings \(x\) and single letters \(a\). Here \(q\) is a state.

A student asked me, why did we define it this way? Why couldn’t we have defined an extended transition function \(\delta'\) recursively by \(\delta'(q, \epsilon) = q\) and \(\delta'(q, ax) = \delta'(\delta(q, a), x)\) for all strings \(x\) and single letters \(a\) instead?

The answer is that we certainly could have made this alternate choice, and it would give the same function!

Prove this. That is, using the above definitions for \(\delta^*\) and \(\delta'\), give a complete formal proof that \(\delta^*(q, x) = \delta'(q, x)\) for all states \(q\) and strings \(x\). Justify each line of your proof adequately.