1. [10 marks]
Which of the following claims are true? Just answer “true” or “false” for each one. No justification necessary.

(a) [2 marks] If \( L \) is a language, then \( \emptyset L = \emptyset \).

(b) [3 marks] \((a^3)^* = (a^*)^3\).

(c) [2 marks] \(a^n b^n c^n = (abc)^n\) for all \( n \geq 0 \).

(d) [3 marks] \(a(ba)^*b = (ab)^*ab\).

2. [10 marks]
Recall the recursive definition of the DFA extended transition function: \( \delta^*(q, \epsilon) = q \), and \( \delta^*(q, xa) = \delta(\delta^*(q, x), a) \), for \( q \) a state, \( x \) a string, and \( a \) a single letter.

Give a formal proof of the following property of the DFA extended transition function: for all states \( q \), and all strings \( x, y \), we have \( \delta^*(q, xy) = \delta^*(\delta^*(q, x), y) \).

Hints: use induction, and say precisely what you are inducting on. You can use the fact that concatenation is associative. Be sure to distinguish carefully between \( \delta \) and \( \delta^* \) in your proof.

3. [10 marks]
(a) [3 marks] Write \((a \cup b)^*\) as a regular expression without using the symbol \( \cup \) (union) anywhere.

(b) [7 marks] A regular expression \( r \) is said to be in \textit{disjunctive normal form} if it looks like \( r = r_1 \cup r_2 \cup \cdots \cup r_n \) for some \( n \geq 1 \), where none of the regular expressions \( r_1, r_2, \ldots, r_n \) contain the symbol \( \cup \) (union). For example, the regular expression \( a^*b^* \cup (ab)^* \cup (c(acb)^*)^* \) is in disjunctive normal form, but \((a \cup b)^*\) is not. Prove that every regular expression can be written in disjunctive normal form.