University of Waterloo
CS 360 — Introduction to the Theory of Computing
Fall 2017
Problem Set 2

Handed out Thursday, September 14, 2017

Due Thursday, September 21, 2017, at 5 PM. Submit to LEARN.

1. [10 marks]
Which of the following claims are true? Just answer "true" or "false" for each one. No justification necessary.

   (a) [2 marks] If L is a language, then $L\emptyset = \emptyset$.

   (b) [3 marks] $(\mathtt{a}^2)^* = (\mathtt{a}^*)^2$.

   (c) [2 marks] $\mathtt{a}^n\mathtt{b}^n = (\mathtt{ab})^n$ for all n

   (d) [3 marks] $\{\mathtt{a}, \mathtt{b}\}^* = (\mathtt{a}^*\mathtt{b}^*)^*$.

2. [10 marks]
Give a regular expression, using only the operators union ($\cup$), concatenation, and Kleene *, for each of the following. Give a brief English explanation of your solution. No formal proof necessary. Try to make your solutions as short as you can.

   (a) [3 marks] $\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^* - \mathtt{a}^*\mathtt{b}^*\mathtt{c}^*$.

   (b) [3 marks] The set of strings over $\{\mathtt{a}, \mathtt{b}\}$ that do not contain two (or more) consecutive occurrences of the same letter.

   (c) [4 marks] The set of strings over $\{\mathtt{a}, \mathtt{b}\}$ containing exactly one pair of consecutive $\mathtt{a}$'s and exactly one pair of consecutive $\mathtt{b}$'s.

3. [10 marks]
Give a formal proof of the identity $|xy| = |x| + |y|$ for strings x, y. Hint: use induction, and be sure to say precisely what you are inducting on.

   You can use the following recursive definition of length of a string: $|\epsilon| = 0$, and for c a single symbol we have $|xc| = |x| + 1$.

4. [Extra credit only; 5 marks]
The current record for the Post problem on strings mentioned in Lecture #2 (see the course home page for more info) seems to be a string of length 43 that achieves 20858070 different strings until a repetition occurs. Find an example, of length ≤ 500, that results in an even larger number of strings. Explain how you did it.