1. [10 marks]
Which of the following claims are true? Just answer “true” or “false” for each one. No justification necessary.

(a) [2 marks] If \( L \) is a language, then \( L\emptyset = \emptyset \).
(b) [3 marks] \((a^2)^* = (a^*)^2\).
(c) [2 marks] \(a^n b^n = (ab)^n\) for all \(n\)
(d) [3 marks] \(\{a, b\}^* = (a^*b^*)^*\).

2. [10 marks]
Give a regular expression, using only the operators union (\( \cup \)), concatenation, and Kleene *, for each of the following. Give a brief English explanation of your solution. No formal proof necessary. Try to make your solutions as short as you can.

(a) [3 marks] \(\{a, b, c\}^* - a^*b^*c^*\).
(b) [3 marks] The set of strings over \(\{a, b\}\) that do not contain two (or more) consecutive occurrences of the same letter.
(c) [4 marks] The set of strings over \(\{a, b\}\) containing exactly one pair of consecutive a’s and exactly one pair of consecutive b’s.

3. [10 marks]
Give a formal proof of the identity \(|xy| = |x| + |y|\) for strings \(x, y\). Hint: use induction, and be sure to say precisely what you are inducting on.

You can use the following recursive definition of length of a string: \(|\epsilon| = 0\), and for \(c\) a single symbol we have \(|xc| = |x| + 1\).

4. [Extra credit only; 5 marks]
The current record for the Post problem on strings mentioned in Lecture #2 (see the course home page for more info) seems to be a string of length 43 that achieves 20858070 different strings until a repetition occurs. Find an example, of length \(\leq 500\), that results in an even larger number of strings. Explain how you did it.