1. [10 marks] Something is wrong with the following proof that the class of regular languages is closed under the operation of Kleene star! Identify the exact line that is false, and explain why it is false.

   1. “We will prove that if \( L \) is regular, then so is \( L^* \).
   2. The definition of \( L^* \) is \( L^* = \bigcup_{i \geq 0} L^i \).
   3. Since the class of regular languages is closed under the concatenation operation,
   4. an easy induction on \( i \) proves that if \( L \) is regular, then so is \( L^i \), for all \( i \geq 0 \).
   5. Now the class of regular languages is closed under the union operation,
   6. so it follows that \( \bigcup_{i \geq 0} L^i \) is also regular.”

2. [10 marks] Use the construction given in class (or in the notes) to convert the following NFA to an equivalent DFA, and draw the result. Do not include any states that cannot be reached from the initial state \( q_0 \).

3. [10 marks] Let \( L \subseteq \Sigma^* \) be a language. Define
   \[
   \text{pref}(L) = \{ x \in \Sigma^* : \text{there exists } y \in L \text{ such that } x \text{ is a prefix of } y \}.
   \]
   Prove that if \( L \) is regular, then so is \( \text{pref}(L) \). Hint: take a DFA \( M \) for \( L \) and modify it so that it becomes a DFA \( M' \) for \( \text{pref}(L) \). Then prove that your DFA \( M' \) has the desired property.