This problem set covers the material of Week 3 on the course web page. You should watch all the videos (or read all the slides) for Week 3 before you try the problems here.

1. [10 marks] Using the pumping lemma, prove that the language \{a^{3n} : n \geq 0\} is not regular.

2. [10 marks] Give an example, with justification, of
   (a) [5 marks] a nonregular language \(L_1\) and an infinite regular language \(L_2\) such that \(L_1L_2\) is regular;
   (b) [5 marks] a nonregular language \(L_1\) and an infinite regular language \(L_2\) such that \(L_1L_2\) is nonregular.

3. [10 marks]
   Suppose we define an operation on nonempty strings, as follows: \(\text{odd}(x)\) deletes all symbols of \(x\) occurring at even-numbered positions. More precisely, for \(n \geq 0\) we have \(\text{odd}(a_1a_2\cdots a_{2n+1}) = \text{odd}(a_1a_2\cdots a_{2n+2}) = a_1a_3a_5\cdots a_{2n+1}\).
   Thus, for example, \(\text{odd} \text{blackberry} = \text{baker}\).
   Extend this to languages in the obvious way: \(\text{odd}(L) = \{\text{odd}(x) : x \in L \setminus \{\epsilon\}\}\).
   Prove that if \(L\) is regular then so is \(\text{odd}(L)\).
   Hint: start with a DFA for \(L\) and modify it somehow, getting an \(\epsilon\)-NFA for \(\text{odd}(L)\). In addition to giving your construction in detail, be sure to give a proof that it actually works.
   This one is likely to be somewhat hard.