1. [10 marks] Use a closure property of the regular languages to prove that
\[ \{ a^p : p \text{ is a prime number} \} \cup \{ a^n b^n : n \geq 0 \} \]
is not regular.

2. [10 marks] Consider the DFA \( M \) given in Week 2’s slides entitled “Deterministic Finite automata”, slide 25:

Using the state elimination method discussed in Week 2, find a regular expression specifying \( L(M) \). Show your work.

3. [10 marks] Describe, as explicitly as possible, a “black-box” decision algorithm for the following problem: given DFA’s \( M_1 \) and \( M_2 \) as black boxes, of \( m \) and \( n \) states, respectively, recognizing \( L_1 \) and \( L_2 \), decide if \( L_1 \subseteq L_2 \).

Remember that (a) you know \( m \) and \( n \) and (b) the only thing you are allowed to do is to input strings to \( M_1 \) and \( M_2 \) and observe whether they accept or reject.

Say explicitly which strings you will input. Justify.