1. [10 marks + 2 marks extra credit possible] A string \( x \) is a palindrome if \( x = x^R \). Give a context-free grammar \( G = (V, \Sigma, P, S) \) that generates all the non-palindromes over the alphabet \( \{a, b\} \). Describe explicitly all four parts: \( V, \Sigma, P, S \), and explain your construction in words. You do not need to give a complete formal proof of correctness.

You will get 2 extra-credit marks if your grammar is unambiguous (and you explain correctly why it is).

2. [10 marks] Develop a black-box algorithm to check, given an NFA \( M = (Q, \Sigma, \delta, q_0, F) \) whether \( L(M) = \Sigma^* \). The only information provided to you is
   - \( \Sigma \), the input alphabet;
   - \( n = |Q| \), the number of states in \( M \);
   - a “black box” \( B \) implementing \( M \), which can only by used by entering a string \( x \) as input, and observing whether \( B \) says “accept” or “does not accept” (corresponding to, respectively, \( x \in L(M) \) and \( x \not\in L(M) \)). You can use \( B \) any (finite) number of times.

In particular, you have no information about \( Q \) except its size, and no information about \( \delta \) or \( F \).

Explain your algorithm, argue it is correct, and give an upper bound on the size of the largest string you have to test, as a function of \( n \).

3. [10 marks] Let \( A, B \) be regular languages such that \( A \subset B \) and \( B - A \) is infinite. Show that there exists a regular language \( L \) that “lies infinitely between \( A \) and \( B \)”, that is, \( A \subset L \subset B \) and both \( B - L \) and \( L - A \) are infinite.

Hint: apply the pumping lemma to \( B - A \).