1. [10 marks] We discussed a “shifting” subroutine in class as part of the way we could simulate a two-way tape TM with a one-way tape TM.

Give the transition diagram of a 1-tape deterministic TM implementing shifting. More precisely, assuming the TM starts in the configuration

$$(q_0, \$) \overline{a_1 \cdots a_n} x,$$

after running the shifter once the new configuration should be

$$(q_{\text{acc}}, \$) \overline{a_n a_{n-1} \cdots a_1} x.$$  

Here $\$ is a special new symbol, not anywhere else on the tape and $\omega$ is a blank, and $x \in \{a, b\}^+$, and $n \geq 0$.

Draw the transition diagram and give full details.

2. [10 marks] Design a 1-tape deterministic Turing machine to reverse its input. That is, from the starting configuration $(q_0, \omega) a_1 a_2 \cdots a_n$, the TM should end in the configuration $(q_{\text{acc}}, \omega) a_n a_{n-1} \cdots a_1$. You can assume that the input alphabet is $\Sigma = \{a, b\}$.

Draw the transition diagram and give full details.

3. [10 marks] Suppose that $L$ is a language that is Turing-recognizable but not Turing-decidable. Show that any TM recognizing $L$ must fail to halt on infinitely many different inputs.