The diagram below shows an extensible hash table with four hash buckets. Each number \( x \) in the buckets represents an entry for a record for which the hashed key value is \( x \). For example, the number 20 in the first hash bucket represents a record for which the hashed key value is 20. The maximum number of records per bucket is four.

a. (2 marks) Give a sequence of two insertions that will cause exactly one new hash bucket to be added without changing the size of the directory. An example of an insertion is ‘insert 9’, which refers to insertion of a tuple with a hashed key value of 9. The two hashed key values in your sequence should be distinct, and should be distinct from the values already shown in the hash table.

Both insertions should be into the bucket pointed to by directory entries 00 and 10, and at least one should have hash value 10. For example:

- insert 8
- insert 10

b. (2 marks) Give a sequence of two insertions that will not cause the hash table’s directory to grow and that will not cause any new hash buckets to be added. Start from the original hash table shown in the figure above, not from the hash table that would result from the inserts from part (a). The two hashed key values in your sequence should be distinct, and should be distinct from the values already shown in the hash table.

One insertion must go to bucket 00 (or 10), and the other insertion must go to bucket 11. For example:

- insert 8
- insert 11

c. (2 marks) Give a sequence of two insertions that will cause the hash table’s directory to double twice. If this is not possible, write NOT POSSIBLE. Start from the original hash table shown in the figure above. The two hashed key values in your sequence should be distinct, and should be distinct from the values already shown in the hash table.

To cause the directory to double twice, both insertions must go into what will become bucket 101 after the directory doubles for the first time. In other words \( x \mod 8 \) must equal 5. For example:

- insert 29
- insert 37
2. (6 total marks)
Suppose that a B+Tree index has been created for some attribute $R.x$ of a relation $R$.

a. (1 mark)
Assume that the B+Tree is clustered and of Type I (index leaves contain tuples of $R$). Consider the two rightmost leaf blocks of the B+Tree. Is it necessarily true that these two blocks are sequential in the file or disk that contains the B+Tree? Answer YES or NO.

No.

b. (2 marks)
Again, assume that the B+Tree is clustered and of Type I. Suppose that $x_1$, $x_2$, and $x_3$ are values of $R.x$, and $x_1 < x_2 < x_3$. Suppose also that the tuples corresponding to $x_1$ and $x_3$ are located on the same page, $p$. Is it necessarily true that the tuple corresponding to $x_2$ is also located on page $p$? Answer YES or NO.

Yes.

c. (2 marks)
Repeat question (b), but this time under the assumption that the index is unclustered and of Type II (index leaves contain tuple IDs).

No.

d. (1 mark) Which of these two types (Type I clustered, Type II unclustered) of indexes will allow the tuples of $R$ to be retrieved in order of their $R.x$ values? Answer "Type I clustered", "Type II unclustered", "both", or "neither".

Both.

3. (12 total marks)

For this question, consider a query that performs a join of tables $R$ and $S$ with the join condition $R.a = S.b$. $R.a$ is the primary key of $R$, and there is a clustered index on $R.a$. $S.b$ is a candidate key of $S$, but there is no index on $S.b$. Suppose that the size of $R$ is $B$ blocks, that the size of $S$ is also $B$ blocks, and that the size of the index on $R.a$ is negligible (much less than $B$).

a. (4 marks)
Suppose the plan for the query is a block nested loop join of $R$ and $S$, with $S$ as the outer. $M$ blocks of memory are available for the join operation, where $M < B$. Give an expression (in terms of $B$ and $M$) for the I/O cost of this plan. Assume that the plan output is not materialized.

$S$ will be read once at a cost of $B$. $R$ will be read $B/M$ times, at a cost of $B$ each time. So, the total cost is:

$$B + \frac{B}{M}B = B + \frac{B^2}{M}$$

b. (2 marks)
Suppose instead that the plan for the query is a merge join of $R$ and $S$. Tuples of $R$ are read in $R.a$ order using the clustered index on $R.a$. $S$ is sorted using an external merge sort, the output of which is pipelined into the join. A total of $M$ blocks of memory are available, where $\sqrt{B} < M < B$. How much memory should be used for the merge join operator, and how much should be used for the sort operator? Briefly justify your answer.

Since $R.a$ is a key of $R$ and $S.b$ is a candidate key of $S$, there are no duplicate join keys. In that case, the merge join operator only needs to hold one tuple from each relation. Therefore, all, or almost all of the available memory should be given to the sort operator.
3. (cont’d)

c. (4 marks)
Estimate the I/O cost of the plan from part (b), assuming that the M blocks of memory are allocated as you specified in your answer to that part. Assume that block writes and block reads have the same cost and that the plan output is not materialized.

For the sort, the run formation phase will involve reading S (cost $B$) and writing $B/M$ sorted runs (cost $B$). Since $\sqrt{B} < M$, the number of runs will be less then $M$, and a single merge pass will suffice. That at will have an I/O cost of $B$ to read in the runs, and no cost to pipeline to the sorted output data. So, the total I/O cost for the sort will be $3B$.

The join will consume the pipelined sort output (no I/O cost) will read $R$ using the index on $R.a$. Since the index is clustered, each block of $R$ will be read once, for a cost of $B$. In addition, some index blocks will have to be read, but this cost is negligible since the index is assumed to be very small.

Thus, the total cost is $4B$: $3B$ from the sort and $B$ from reading $R$.

d. (2 marks)
Assuming that $\sqrt{B} < M < B$, under what circumstances is the block nested loop join plan preferable to the merge join plan? Express your answer in terms of $B$ and $M$, and briefly justify it.

The block nested loop join is preferable when

$$B + \frac{B^2}{M} < 4B$$

which occurs when $\frac{B}{M} < M$.
The pseudo-code below implements the \texttt{GetNext} method of an iterator version of tuple-oriented nested-loop join, as discussed in class.

\begin{verbatim}
Iterator state:
Iterator R; // right (inner) child
Iterator L; // left (outer) child
Tuple t; // current outer tuple

GetNext() {
    tuple t2; // current inner tuple
    if (t == NULL) t = L.GetNext();
    while (t <> NULL) {
        while((t2 = R.GetNext()) <> NULL) {
            if (t matches t2) return (t join t2)
        }
        R.Close();
        R.Open();
        t = L.GetNext();
    }
    return(NULL);
}
\end{verbatim}

Implement the \texttt{GetNext} method of an iterator for merge join, assuming that both the left and right inputs are already sorted on their join keys. Declare the iterator state and write your code in the style of the example above. To simplify this problem, you may assume that the \textit{left input} (only) does not contain duplicate join keys. The right input may contain duplicates.

\begin{verbatim}
Iterator state:
Iterator R; // right (inner) child
Iterator L; // left (outer) child
Tuple tl; // current left tuple (need to save, as it may match multiple right tuples)

// on exit, either tl has matched tr or one or both inputs are exhausted
GetNext() {
    Tuple tr; // right tuple
    // don't get a new tl except when GetNext is first called
    if (tl == NULL) tl = L.GetNext();
    tr = R.GetNext();
    while (tl <> NULL and tr <> NULL) {
        if (tl matches tr) then return (tl join tr);
        if (joinkey(tl) < joinkey(tr)) then
            tl = L.GetNext()
        else
            tr = R.GetNext()
    }
    return(NULL)
}
\end{verbatim}
5 (8 total marks) Consider the hybrid hash join algorithm presented in the notes and discussed class. Assume that a total of $M$ blocks of memory are available for the join, of which $k$ blocks are used to build the hash table for the first partition and the remaining $M-k$ blocks are used to stage the remaining partitions to temporary files. For the purposes of this question, make the simplifying assumption that the hash table is perfectly space-efficient, i.e., $k$ blocks worth of tuples can be stored in a hash table of size $k$ blocks.

a. (4 marks)
What is the size of the largest build relation that can be handled by this join operator without having to recursively re-partition the build relation? Express your answer in terms of $M$ and $k$, and justify your answer.

Each partition must have at most $k$ blocks, since it needs to fit into the hash table. The maximum number of partitions is $M-k$ (actually, $M-k+1$), since we need one block to stage each partition that is being sent to a temporary file on the disk. Thus, the largest build file that can be handled is about

$$k(M-k) = Mk - k^2$$

b. (2 marks)
What is the best value for $k$, if the goal is to maximize the size of build table that can be handled by this operator without re-partitioning? Justify your answer.

$$\frac{d(Mk - k^2)}{dk} = M - 2k$$

This reaches zero when $k = M/2$, which maximizes build file size.

c. (2 marks)
Does memory size impose a limit on the maximum size of the probe input that can be handled (without re-partitioning) by this operator? If so, what is the maximum probe table input size (in terms of $M$ and $k$) that can be handled by this operator? If not, write NO LIMIT.

NO LIMIT
Memory imposes a limit on the number of partitions of the probe relation (which will be $M-k+1$), but not on the size of the partitions, since they do not need to be loaded into the in-memory hash table.
6 (10 total marks) Provide brief answers to the each of the following questions. “Brief” means a sentence or two.

a. (2 marks)
What does pipelining mean, in the context of relational query processing?

Pipelining means that each query operator makes its output tuples available to the next operator as soon as they are produced, rather than waiting until it has finished producing all of its output tuples.

b. (2 marks)
What is the database physical design problem?

It is the problem if choosing which indexes (or other physical structures) to create for a particular database, usually given some information about the types of queries and updates that are expected.

c. (2 marks)
An external merge sort operates in two phases. What are the two phases, and what occurs in each phase?

The first phase is run formation, during which the entire input is consumed and several individually-sorted runs are produced. The second phase is merging, during which multiple sorted runs are merged to produce longer runs - and eventually one single sorted run.

d. (2 marks)
How does the generalized clock replacement algorithm differ from the basic clock algorithm?

The basic algorithm maintains one bit per cached page to indicate whether that page has been used recently, and tries to evict pages that have not been used recently. The generalized algorithm instead maintains a counter per cached page and tries to evict pages with lower counter values.

e. (2 marks)
What is the cache inclusion problem?

In a two-level cache hierarchy, cache inclusion refers to the problem that the lower level cache may contain many of the same pages that are found in the upper level cache.