let \( X, Y \) nonempty word such that \( X, Y \) don't prefix each other (eg: \( X = 1 \), \( Y = 0 \))
define infinite sequence \( \{ T_k \} \) where \( T_k = \begin{cases} X & \text{if } k = 1 \\ (T_k)^k Y & \text{if } k > 1 \end{cases} \)
easy to see that \( T_i \) prefix \( T_j \) when \( i \leq j \)
given arbitrary \( \text{int } e \geq 2 \) , \( \text{int } p \geq 1 \) (given arbitrary position \( p \) and power \( e \))
let \( q = \min \{ m | |T_m| > p \} \) // \( T_q \) include position \( p \)
if \( q \leq e \) : let \( U = T_e [1, p-1] \), \( V = T_q \cdot T_e [1, e+1] \) ( \( T_e \) includes position \( p \) since \( e \geq q \) )
\[ T_{e+1} = (UV)^{e+1} Y \Rightarrow U^* T_{e+1} = (UV)^e V Y \text{ as wanted} \]
otherwise \( q > e \) : let \( U = T_q [1, p-1] \), \( V = T_q \cdot T_q [1, q] \)
( \( T_e \) can only handle limited size of \( p \), therefore use longer \( T_q \) when position \( p \) exceed \( T_q \) )
\[ T_{q+1} = (UV)^{q+1} Y \Rightarrow U^* T_{q+1} = (UV)^q V Y \text{ as wanted} \]
therefore, \( T_{\max(qe)+1} \) has a power of \( e \) start at position \( p \)
assume \( Tw \) is periodic, let \( z \) be the shortest period, hence \( Tw = z^w \)
since \( T_{121} \) prefix \( Tw \), then \( T_{121} = (T_{121})^* Y = z^w Y \)
by definition of \( T \), \( X \) prefix \( T_i \) \( \forall i \), hence \( X \) prefix \( z^w \)
if \( |Y| \leq |X| \) : since \( Y \) not prefix \( X \), \( Y \) not prefix \( z^w \)
  hence \( T_{121} = z^{12 |12|} Y \) not prefix \( z^w \)
  else \( |Y| > |X| \) : since \( X \) not prefix \( Y \), \( Y, X \) mismatch in first \( |X| \) bits
  hence \( Y \) not prefix \( z^w \), \( T_{121} = z^{12 |12|} Y \) not prefix \( z^w \)

\( T_{121} \) not prefix of \( z^w \) contradiction with assumption ,
Therefore, \( Tw \) is aperiodic

hence, \( T_w \) which is an aperiodic infinite word, has an arbitrary large power at arbitrary position.