Theorem 1. There exists an infinite word over a 3 letter alphabet avoiding the pattern $xx'$ where $x'$ is a conjugate of $x$.

Proof. Let $x \in \{0, 1, 2\}^+$ and $x'$ be a conjugate of $x$. Since $x$ and $x'$ are conjugates, there exist words $u$ non-empty and $v$ possibly empty such that $x = uv$ and $x' = vu$. If $v = \epsilon$ then $xx' = u^2$, and if $v \neq \epsilon$ then $xx' = uv^2u$. In both of these cases $xx'$ contains a non-empty square as a factor. But we know that there is an infinite word over a 3 letter alphabet that avoids squares (Theorem 2.5.2). Therefore, there exists an infinite word over a 3 letter alphabet avoiding the pattern $xx'$ where $x'$ is a conjugate of $x$. \qed