Problem 18:
Is the following language regular? \( L = \{ xwx^R : x, w \in \{0, 1\}^+ \} \).

I claim that \( L \) is regular, as it is the language of a regular expression. By Kleene’s Theorem, the language of a regular expression is a regular language.

Consider the following regular expression: \( R = (0\{0, 1\}^+0) \cup (1\{0, 1\}^+1) \).

I claim that \( L \) is the language of \( R \).
In other words, I claim that \( L = L(R) \), the language of \( R \), is equal to \( L \).
To prove that \( L(R) = L \), we will use set equality.
We will first prove that \( L(R) \subseteq L \), and then prove that \( L \subseteq L(R) \).

\( L(R) \subseteq L \):
Let \( y \) be some word in \( L(R) \).
Then \( y = ava \), for some letter \( a \) where either \( a = 0 \) or \( a = 1 \), and some word \( v \in \{0, 1\}^+ \).
Construct words \( x \) and \( w \), such that \( x = a \) and \( w = v \).
Since \( |x| = 1 \), \( x = x^R \).
Then \( y = xwx^R \), and \( x, w \in \{0, 1\}^+ \). So every \( y \) is in \( L \), by the definition of \( L \).
Therefore, \( L(R) \subseteq L \).

\( L \subseteq L(R) \):
Let \( z \) be some word in \( L \).
By definition of \( L \), \( z = xwx^R \), where \( x, w \in \{0, 1\}^+ \).
Since \( x \neq \epsilon \) by definition, let \( x = ax' \), where \( a \) is the first letter of \( x \) (either 0 or 1).
For any possible \( x' \), we have that \( x' \in \{0, 1\}^* \).
Then \( z = xwx^R = ax'w(x')^Ra = a\{0, 1\}^*\{0, 1\}^+\{0, 1\}^*a = a\{0, 1\}^+a \).
\( R \) clearly accepts every possible \( z \), whether \( a = 0 \) or \( a = 1 \).
Therefore, \( L \subseteq L(R) \).

Since \( L(R) \subseteq L \), and \( L \subseteq L(R) \), it must be that \( L = L(R) \).
\( L(R) \) is regular by Kleene’s Theorem. Therefore, \( L \) is a regular language. \( \square \)