29. Two automata $M_1, M_2$ over the same input alphabet are *isomorphic* if there is some permutation of the names of the states that changes $M_1$ into $M_2$. Show that two minimal NFAs accepting the same language are not necessarily isomorphic.

Consider the regular language $L$ defined by $00^*$ over $\Sigma = \{0\}$, with two NFAs that accept the language $M_1 = (Q_1, \Sigma, \delta_1, p_1, F_1), M_2 = (Q_2, \Sigma, \delta_2, p_2, F_2)$ as follows:

- $Q_1 = \{p_1, q_1\}$
- $F_1 = \{q_1\}$
- $\delta_1(p_1, 0) = \{p_1, q_1\}$
- $\delta_1(q_1, 0) = \{q_1\}$

- $Q_2 = \{p_2, q_2\}$
- $F_2 = \{q_2\}$
- $\delta_2(p_2, 0) = \{q_2\}$
- $\delta_2(q_2, 0) = \{q_2\}$

So $M_1$ appears as so:

```
\[
\begin{array}{c}
\text{start} \\
\rightarrow \quad p_1 \\
\quad \quad 0 \\
\quad \quad q_1 \\
\end{array}
\]
```

which accepts $L$ because we can take any number of zeros, followed by one zero ($0^*0$, which is the same as $00^*$). $M_2$ appears as so:

```
\[
\begin{array}{c}
\text{start} \\
\rightarrow \quad p_1 \\
\quad \quad 0 \\
\quad \quad q_1 \\
\end{array}
\]
```

which accepts one zero followed by any number of zeros ($00^*$).

If we were to construct a one-state NFA accepting $L$, either:

- The start state would be an accepting state, in which case the NFA would accept $\varepsilon$ (which is not in $L$), or

- The start state would be an accepting state, in which case there would be no accepting states. So the NFA would accept nothing.

Therefore a two-state NFA for $L$, if it exists, is minimal. Specifically, $M_1$ and $M_2$ are minimal.

Finally, $p_1$ and $p_2$ start states do not have the same number of transitions. Since the start states must be mapped to each other in an isomorphism, $M_1$ and $M_2$ are not isomorphic. □