Theorem 1. Let \( x \) and \( y \) be two non-empty antipalindromes. Then \( xy \) is an antipalindrome if and only if there exist a non-empty word \( w \) and integers \( n, m \geq 1 \) such that \( x = w^n \) and \( y = w^m \).

Proof. Suppose \( x \) and \( y \) are two non-empty antipalindromes.

By definition, \( xy \) is an antipalindrome if and only if

\[
xy = (xy)^R \\
\iff \overline{x} y^R = y^R \overline{x} \\
\iff \overline{y} x = xy \\
\iff yx = xy
\]

By the 2nd Lyndon-Schutzenberger theorem, this occurs if and only if there exist a non-empty word \( w \), and integers \( n, m \geq 1 \) such that \( x = w^n \) and \( y = w^m \). \( \square \)