30. What are the Myhill-Nerode equivalence classes of the language \( \{a^m b^n : 1 \leq m \leq n \} \)?

\( L = \{a^m b^n : 1 \leq m \leq n \} \) is not a regular language, so there is an infinite number of equivalence classes for it. Specifically, they are \( \{[\epsilon], [a], [a^2], ..., [a^n], [a^n b], ..., [a^2 b], [a b], [b] \} \), where \( n \) is arbitrarily large. See below:

![Transition diagram](image)

All transitions not pictured lead to the failure state, \( [b] \).

To prove that these are all unique equivalence classes, we provide for each pair of equivalence classes \( x, y \) a word \( z \) such that \( xz \in L \) but \( yz \notin L \).

For all pairs with the form \( a^i \) and \( a^j \), \( 1 \leq i < j \), we take \( z = b^j \). In this case, \( xz = a^i b^j \in L \), but \( yz = a^i b^i \notin L \).

For all pairs with the form \( a^i b \) and \( a^j b \), \( 1 \leq i < j \), we take \( z = b^{i-1} \). In this case, \( xz = a^i b^i \in L \), but \( yz = a^i b^i \notin L \).

For all pairs with the form \( a^i \) and \( a^i b \), we take \( z = ab^{i+1} \). In this case, \( xz = a^{i+1} b^{i+1} \in L \), but \( yz = a^i b a b^{i+1} \notin L \).

For all pairs where one element \( (x) \) is the equivalence class of the empty string, we can simply take \( z = ab \) so that \( xz = ab \in L \), but any \( yz \) would be of the incorrect form, or would have \( m > n \), and would thus not be in \( L \).

Similarly, for pairs where one element \( (y) \) is the equivalence class of the failure state \( [b] \), we could find a \( z \) such that \( xz \in L \), whereas any string appended to \( [b] \) would not be in the language.

After all this, we can see that these are all the unique equivalence classes of \( L \), and that there is an infinite number of them.