32. What are the Myhill-Nerode equivalence classes for the language \( \{a^{2^n} : n \geq 0\} \)

**solution**

Any word \( a^x \) is in a class with the unique element.

To prove this, we need to show there exists no \( y \) s.t. \( x + z = 2^n \) for some \( n \) if \( y + z = 2^m \) for some \( m \).

If there exists such \( y \),

WLOG, let \( y < x \).

\[ \forall n \geq \log_2 x, \exists z, \exists m, \ x + z = 2^n, y + z = 2^m \]

\[ \Rightarrow \forall n \geq \log_2 x, \exists m < n, \ x - y = 2^n - 2^m \]

Let \( n = n_0 = \lceil \log_2 (x - y) \rceil + 1 \), then \( 2^{n_0} - 2^m \geq 2^{n_0 - 1} \neq x - y \).

Contradiction.