1. **Question** Consider the operation $\text{del}_a(w)$ that deletes all letters $a$ from the word $w$, and extend the operation to languages in the obvious way. Give an example of a class of regular languages $L$, accepted by DFA’s with $O(n)$ states, such that the smallest DFA accepting $\text{del}_a(L)$ needs at least $\Omega(2^n)$ states.

**Solution:**

Let $L_n = \{\{0,1\}^*21\{0,1\}^{n-1}\}$

Let $M_n = (Q, \Sigma, \delta, q_0, F)$ be the DFA for $L_n$ with:

- $Q = \{q_0, q_1, ..., q_{n+1}\}$
- $\delta(q_0, \{0,1\}) = q_0$
- $\delta(q_0, 2) = q_1$
- $\delta(q_1, 1) = q_2$
- $\delta(q_i, \{0,1\}) = q_{i+1}$ for $2 \leq i \leq n$
- $F = \{q_{n+1}\}$

It is clear $M_n$ has $O(n)$ states.

$\text{del}_2(L_n) = \{\{0,1\}^*1\{0,1\}^{n-1}\}$ ie. the language where the $n^{th}$ character from the right is a 1, and we’ve proved in class that this language’s minimal DFA has $2^n$ states. (Theorem 3.9.6 in the textbook)