7. Give an example of a unary nonregular language L, not containing ϵ, for which L^2 is regular. Our language is $L = \{x \text{ such that } |x| \text{ is odd, or is a power of } 2\}$.

Firstly we must show that L is non-regular. For a contradiction, let us assume that L is regular. Note that the language of odd length strings of 1s, here denoted L' is also regular, as it may be represented as the regular expression $1(11)^*$. Thus, as L and L' are both regular, so to should their relative complement (the set of elements of L which aren't in L'). However, their relative complement, $L - L'$ is the language consisting of all strings of a length that is a power of 2. This language is clearly not regular, contradicting our assumption that L is regular. Thus we have proven that L is nonregular.

Lastly, we must prove that L^2 is regular. We note that 1 and 11 are both elements of L, as well as every string of odd length. We may construct every string of 1s of even length by summing 1 and every odd length string of ones. Similarly we may construct every odd length string of 1s (excluding 1), by suming 11 and every string of odd length. As such, we have shown that L^2 contains every string of even length, and every string of odd length except 1. This language is clearly regular, as it consists of $\Sigma^+ - 1$, which is the difference of two regular languages.

Thus, we have proven that L is nonregular and L^2 is regular, as required.