We are given regular languages $L_2, L_1 \subseteq \Sigma^*$ with $L_2 - L_1$ infinite.

We know that $L_2 - L_1$ is regular and therefore has a DFA $M = (Q, \Sigma, \delta, q_0, F)$ accepting it. Because the language is infinite, there must be a reachable cycle in the DFA starting at $p \in Q$ that leads to an accepting state. That is, there exists strings $x, y, z$ be the strings such that $\delta(q_0, x) = q_i, \delta(p, y) = p$ and $\delta(p, z) \in F$. Thus, the language $\{xy^n z\} \subseteq L_2 - L_1$.

Construct $L = L_1 \cup \{xy^n z : n = 2m\}$. Then $L - L_1 = \{xy^n z : n = 2m\}$ which is infinite. Furthermore, $\{xy^n z\} \subseteq L_2 - L_1 \implies L_2 - L \supseteq \{xy^n z : n = 2m + 1\}$ which is also infinite.