Problem session 5

Problem 5

Let $p, q$ be primes such that $p \neq q$. Let $x = 0^p$ and $y = 0^q$.

Claim: $x \not\equiv y$.

Let $z = 1^p$. Then $xz \not\in L$ because $\gcd(p, p) = p$, but, $yz \in L$ since $\gcd(p, q) = 1$.

Hence, $x$ and $y$ belong to distinct equivalence classes of $R_L$.

By the claim, $R_L$ has infinitely many equivalence classes because there are infinitely many prime numbers. Thus, $L$ is not regular.