What are the Myhill-Nerode equivalence classes for the language $L = \{ a^{2^n} : n \geq 0 \}$ (over $\Sigma = \{a\}$)? Each word in $\Sigma^*$ is in its own equivalence class.

**Proof.** Suppose for a contradiction that there are distinct words $x$ and $y$ such that $x E y$. So $xz \in L \iff yz \in L$ for all $z \in \Sigma^*$. Without loss of generality, we may assume $|x| < |y|$.

Let $z$ be the word such that $|xz| = 2^n$ for some $n \geq 0$, and hence $xz \in L$. This implies that $yz \in L$, in particular, $|yz| = 2^m$ for some $m > n$.

Now, consider $z' = zxz$:

$|xz'| = |zxzxz| = 2|xz| = 2^{n+1}$, so $xz' \in L$.

However, $2^m < |yz'| = |yzzxz| < 2|yz| = 2^{m+1}$. It is not possible for $yz'$ to be in $L$, since its length is strictly between $2^m$ and $2^{m+1}$. So it is not true that $xz \in L \iff yz \in L$ for all $z \in \Sigma^*$, and we have the desired contradiction. \qed