Prove that a string $x$ is an antipalindrome if and only if there exists $y$ such that 

$x = yy^R$

Proof.

$\implies$

There exists a $y$ such that $x = yy^R$.

Consider $\bar{x} = y^Ry$, and

$x^R = (yy^R)^R$

$= (y^R)^Ry^R$

$= yy^R$

So, $\bar{x} = x^R$. Thus, $x$ is an antipalindrome.

$\impliedby$

Let $x$ be an antipalindrome. So $x^R = \bar{x}$. Let $n = |x|$.

If $n$ is odd, then $n = 2k + 1$ for some $k$.

Consider the character in position $k + 1$ of the strings $x^R$ and $\bar{x}$.

$x^R[k + 1] = x[n - (k + 1) + 1]$

However, since $x^R = \bar{x}$, but $x^R[k + 1] \neq \bar{x}[k + 1]$, this is a contradiction.

So, $n$ is even.

Since $x^R = \bar{x}$,

$x[i] = x[n - i + 1]$, for $1 \leq i \leq \frac{n}{2}$ \hspace{1cm} (1)

Let $y = x[1...\frac{n}{2}]$

Note that $y = x[\frac{n}{2}]\cdot x[\frac{n}{2} - 1]...x[0 + 1]$, by (1)

so $y^R = x[0 + \frac{1}{2}]...x[\frac{n}{2}]$

and then $y^R = x[0 + \frac{1}{2}]...x[n]$. So finally, we have

$x = x[1...\frac{n}{2}]\cdot x[\frac{n}{2} + 1...n]$

$= yy^R$