Problem 9: Prove or disprove: for all integers $m, n \geq 1$, $x^m$ is an anti-palindrome if and only if $x^n$ is an antipalindrome.

Solution:
We show that for $m \in \mathbb{Z}_{\geq 1}$, $x^m$ is an antipalindrome if and only if $x$ is.

We can draw a picture. Things look like:

\[
\begin{array}{c}
\begin{array}{cccc}
\overline{x}^m \\
\overline{\overline{x}}^m \\
(x^m)^R \\
\end{array}
\end{array}
\]

We know that each of the blocks have the same length since $|x| = |x^R| = |\bar{x}|$. That means that if $\overline{x}^m = (x^m)^R$, then we must have that the first $|x|$ symbols agree and so $\overline{x} = x^R$ so $x$ is an antipalindrome.

Conversely, if $\overline{x}^m \neq (x^m)^R$, then they must differ in the first $|x|$ characters (otherwise, they would have to agree everywhere). Thus, $\overline{x} \neq x^R$. So $x^m$ is an antipalindrome if and only if $x$ is.

To complete the proof, let $m, n \geq 1$ with $m, n \in \mathbb{Z}$. We have:

\[
x^m \text{ antipalindrome } \iff x \text{ antipalindrome } \iff x^n \text{ antipalindrome}
\]

as required.