CS 462 Group Problem-Solving Session
Winter 2018
Session 1

January 6, 2018

Recall that a word x is a palindrome if $x = x^R$, where x^R denotes the reversal of x.

For a word $x \in \{0,1\}^*$, let $\overline{x}$ denote the word obtained by changing each 0 to 1 and vice versa. Call a word x an *antipalindrome* if $\overline{x} = x^R$. Thus, for example, `001011` is an antipalindrome.

Recall that Ш denotes the "perfect shuffle", so that, for example, `clip`Ш`aloe` = `calliope`.

1. Show that x is an even-length palindrome if and only if there exists a string y such that $x = y \,\text{Ш}\, y^R$.

2. Call a language L *commutative* if for all $x, y \in L$ we have $xy = yx$. Show that L is commutative if and only if there exists a word w such that $L \subseteq w^*$.

3. Show that for every infinite string $\mathbf{w}$ there must be some letter a and some finite string x such that axa appears infinitely often as a subword of $\mathbf{w}$. Furthermore such an x exists with $|x| \leq |\Sigma| - 1$, where Σ is the alphabet.

4. Can you construct an aperiodic infinite binary word in which there is a square beginning at every position? Here "aperiodic" means "not ultimately periodic".

5. Harder than the previous one: can you construct an aperiodic infinite word in which there are powers of arbitrarily large exponent beginning at every position? Hint: construct it iteratively.

6. Prove that a string x is an antipalindrome if and only if there exists y such that
$$x = y\,\overline{y^R}.$$

7. Prove or disprove: for all integers $m, n \geq 1$, x^m is an antipalindrome if and only if x^n is an antipalindrome.

8. When is the concatenation of two antipalindromes an antipalindrome? Give necessary and sufficient conditions. Possible strategy: do some experiments.

9. When is the concatenation of two antipalindromes a palindrome? Give necessary and sufficient conditions. Possible strategy: do some experiments.