CS 462 Group Problem-Solving Session
Session 2
January 2018

January 17, 2018

Recall that a word x is a palindrome if $x = x^R$, where x^R denotes the reversal of x.

For a word $x \in \{0,1\}^*$, let $\overline{x}$ denote the word obtained by changing each 0 to 1 and vice versa. Call a word x an *antipalindrome* if $\overline{x} = x^R$. Thus, for example, `001011` is an antipalindrome.

Recall that two words x, y are conjugates if there are words u, v such that $x = uv$ and $y = vu$.

1. Show that two words x, y are conjugates iff there exists a word t such that $xt = ty$. Furthermore, if t exists then we can always find one with $|t| \leq |x|$.
2. A follow-up: show that two words x, y are conjugates iff there exists a word t such that $xt = ty$ or a word s such that $sx = ys$. Furthermore argue that there exists such an s or a t of length $\leq |x|/2$.
3. Is it possible to avoid the pattern xx', where x' is a conjugate of x, over a 3-letter alphabet? (Remember that "avoid the pattern" means "does there exist an infinite word containing no occurrences of the pattern as a subword?")
4. Last week a student constructed an aperiodic infinite binary word in which there are squares beginning at every position. Can you construct an infinite aperiodic binary word where there are infinitely many different squares, that is, *arbitrarily large* squares, beginning at every position?
5. Prove that a string x is an antipalindrome if and only if there exists y such that
$$x = y\,\overline{y^R}.$$
6. Is the Thue-Morse word $\mathbf{t}$ recurrent? That is, if x is a subword of $\mathbf{t}$, must x occur at infinitely many different positions of $\mathbf{t}$?
7. When is the concatenation of two antipalindromes an antipalindrome? Give necessary and sufficient conditions. Possible strategy: do some experiments.
8. When is the concatenation of two antipalindromes a palindrome? Give necessary and sufficient conditions. Possible strategy: do some experiments.