1. Is the following language regular? \( \{ xwR : x, w \in \{0, 1\}^+ \} \).

2. Is the following language regular? \( \{ xx^Rw : x, w \in \{0, 1\}^+ \} \).

3. Find an expression for the perfect shuffle of two languages \( L_1 \) and \( L_2 \) in terms of operations like morphism, inverse morphism, and intersection. Hint: one approach is to modify the expression we found for ordinary shuffle. Conclude that if \( L_1, L_2 \) are regular then so is the perfect shuffle of \( L_1 \) and \( L_2 \).

4. Define \( \text{min}(L) = \{ x \in L : \text{no proper prefix of } x \text{ is in } L \} \).

   Find an expression for \( \text{min}(L) \) in terms of operations like quotient, complement, etc. Conclude that if \( L \) is regular, so is \( \text{min}(L) \).

5. Define \( \text{max}(L) = \{ x \in L : x \text{ is not a proper prefix of any } y \in L \} \).

   Find an expression for \( \text{max}(L) \) in terms of operations like quotient, complement, etc. Conclude that if \( L \) is regular, so is \( \text{max}(L) \).

6. If \( x \) and \( y \) are binary strings, by \( x \lor y \) we mean the bitwise “or” of \( x \) and \( y \). Thus, for example, \( 0011 \lor 1010 = 1011 \). Show that if \( L_1 \) and \( L_2 \) are regular languages over a binary alphabet, that \( L_1 \lor L_2 = \{ x \lor y : x \in L_1, y \in L_2, |x| = |y| \} \) is regular.

7. Suppose \( L \) is regular. Is the language

   \[ \{ xz : \text{there exists } y \text{ such that } xyz \in L \text{ and } |x| = |y| = |z| \} \]

   regular?