1. Define
\[ \text{min}(L) = \{ x \in L : \text{no proper prefix of } x \text{ is in } L \} \].
Find an expression for \( \text{min}(L) \) in terms of operations like quotient, complement, etc. Conclude that if \( L \) is regular, so is \( \text{min}(L) \).

2. Suppose \( L \) is regular. Is the language
\[ \{ xz : \text{there exists } y \text{ such that } xyz \in L \text{ and } |x| = |y| = |z| \} \]
regular?

3. Consider the following transformation on languages:
\[ \sqrt{L} = \{ x \in \Sigma^* : \text{there exists } y \in \Sigma^* \text{ such that } |y| = |x|^2 \text{ and } xy \in L \} \].
Show that if \( L \) is regular, then so is \( \sqrt{L} \). Hint: use the boolean matrix approach and follow the general idea in the proof of \( \log(L) \) we just did in class.

4. Let \( M \) be an NFA. Show that the set of all strings in \( L(M) \) having exactly one accepting path is a regular language. Hint: instead of using Boolean matrix multiplication, use ordinary matrix multiplication to compute the number of accepting paths.

5. Show that if \( M \) is an \( n \)-state DFA, and accepts at least one string that is a palindrome, then it accepts a palindrome of length \( < 2n^2 \).

6. Let \( L_1, L_2 \) be regular languages with \( L_2 - L_1 \) infinite. Show there exists a regular language \( L \) with \( L_1 \subseteq L \subseteq L_2 \) with both \( L_2 - L \) and \( L - L_1 \) infinite. (Here \( A - B \) is set difference, defined to be those strings in \( A \) but not in \( B \).)

7. Let \( L \) be a regular language, and let \( s \) be a substitution by regular languages. Must \( s^{-1}(L) = \{ x : s(x) \subseteq L \} \) be regular?