CS 462 Group Problem-Solving Session
Session 4
Winter 2018

1. Recall that $\sqcup\!\sqcup$ denotes the "perfect shuffle" in which characters exactly alternate, so that, for example, $\texttt{clip} \sqcup\!\sqcup \texttt{aloe} = \texttt{calliope}$. Find an expression for the perfect shuffle of two languages L_1 and L_2 in terms of operations like morphism, inverse morphism, and intersection. Hint: one approach is to modify the expression we found for ordinary shuffle. Conclude that if L_1, L_2 are regular then so is the perfect shuffle of L_1 and L_2.

2. Is the Thue-Morse word closed under reversal of subwords? That is, if x is a subword of $\mathbf{t}$, must x^R also be a subword?

3. Can you construct an aperiodic infinite word in which there are powers of arbitrarily large exponent beginning at every position? Hint: construct it iteratively.

4. Show that if M is an n-state DFA, and accepts at least one string that is a palindrome, then it accepts a palindrome of length $< 2n^2$. Hint: using M design an NFA-ϵ to accept the first halves of the palindromes in $L(M)$.

5. Let L_1, L_2 be regular languages with $L_2 - L_1$ infinite. Show there exists a regular language L with $L_1 \subseteq L \subseteq L_2$ with both $L_2 - L$ and $L - L_1$ infinite. (Here $A - B$ is set difference, defined to be those strings in A but not in B.)

6. Let L be a regular language, and let s be a substitution by regular languages. Must $s^{-1}(L) = \{x \ : s(x) \subseteq L\}$ be regular?

7. Give an example of a unary nonregular language L, not containing ϵ, for which L^2 is regular.

8. Call a language L bounded if there exist a finite number of words $w_1, w_2, \ldots, w_n$ such that $L \subseteq w_1^* \cdots w_n^*$. Give an example of a regular language that is not bounded.