CS 462 Group Problem-Solving Session
Winter 2018
Session 5

1. Define the following transformation on languages, similar to $\log(L)$ in the text, p. 76:

$$\text{sqrt}(L) = \{x \in \Sigma^* \ : \ \exists y \text{ such that } |y| = |x|^2 \text{ and } xy \in L\}.$$

   Show that if L is regular, then so is $\text{sqrt}(L)$. Hint: modify the construction we used for $\log(L)$.

2. Let M be an NFA. Show that the set of all strings in $L(M)$ having exactly one accepting path is a regular language. Hint: instead of using Boolean matrix multiplication, use ordinary matrix multiplication to compute the number of accepting paths.

3. Let L_1, L_2 be regular languages with $L_2 - L_1$ infinite. Show there exists a regular language L with $L_1 \subseteq L \subseteq L_2$ with both $L_2 - L$ and $L - L_1$ infinite. (Here $A - B$ is set difference, defined to be those strings in A but not in B.)

4. Two automata M_1, M_2 over the same input alphabet are said to be *isomorphic* if there is some permutation of the names of the states that changes M_1 into M_2. The Myhill-Nerode theorem says that two minimal DFA's accepting the same regular language are isomorphic. Show, by means of an example, this is not true for NFA's.

5. What are the Myhill-Nerode equivalence classes of the language $\{a^m b^n \ : \ 1 \leq m \leq n\}$?

6. Use the Myhill-Nerode theorem to show that the language

$$\{0^m 1^n \ : \ \gcd(m, n) = 1\}$$

   is not regular. Hint: consider strings of the form 0^p where p is a prime.

7. What are the Myhill-Nerode equivalence classes for the language $\{a^{2^n} \ : \ n \geq 0\}$?