CS 462 Problem-Solving Session
Week 6

1. Show that if a n-state NFA accepts a string w by at least two different acceptance paths, then it accepts such a string with $|w| < n^2 + n$.

2. Describe all the equivalence classes for the Myhill-Nerode equivalence relation for the language $L = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^n \ : \ n \geq 1\}$. Hint: there are a number of cases to consider.

3. Let $L = \{x \in \Sigma^* \ : \ x = x^R\}$, the language of palindromes over the alphabet $\Sigma = \{0, 1\}$. Show that every word of Σ^* is in a Myhill-Nerode equivalence class by itself.

   Hint: This one is slightly tricky: it is easy to show that two different strings w, w' of the same length are in different equivalence classes, but a bit harder when w and w' are of different lengths.

4. Measure the size of a CFG as follows: add up $|\alpha|+1$ for all productions in the grammar of the form $A \to \alpha$. Show that under this measure, the minimal CFG for a language need not be unique.

5. Define the following transformation on languages, similar to $\log(L)$ in the text, p. 76:

   $$\text{sqrt}(L) = \{x \in \Sigma^* \ : \ \exists y \text{ such that } |y| = |x|^2 \text{ and } xy \in L\}.$$

   Show that if L is regular, then so is $\text{sqrt}(L)$. Hint: modify the construction we used for $\log(L)$.

6. Let L_1, L_2 be regular languages with $L_2 - L_1$ infinite. Show there exists a regular language L with $L_1 \subseteq L \subseteq L_2$ with both $L_2 - L$ and $L - L_1$ infinite. (Here $A - B$ is set difference, defined to be those strings in A but not in B.)

7. What are the Myhill-Nerode equivalence classes for the language $\{a^{2^n} \ : \ n \geq 0\}$?