

University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2018
Problem Set 2

Distributed Monday, January 15 2018.

Due Monday, January 22 2018 by 5 PM. Hand in to LEARN.

All answers should be accompanied by proofs. In all problems the underlying alphabet Σ is assumed to be finite.

1. [10 marks] Fix an alphabet Σ . Let $x, z \in \Sigma^*$ be strings. Show that no matter what x, z are, the language $L_{x,z} := \{y \in \Sigma^* : xy = yz\}$ is a regular language.
2. [10 marks] Let w be a nonempty word of length n . Show that w is primitive if and only if w has n distinct conjugates.
3. [10 marks] Call a word w *odd* if every nonempty subword has the property that at least one letter appears an odd number of times. For example, `abac` is odd, but `cabcb` is not.
 - (a) [5 marks] Show that if w is an odd word over an alphabet with k letters, then $|w| < 2^k$. (Hint: it is possible, but surprisingly difficult, to prove this by induction on k . I suggest trying to find a proof *not* using induction.)
 - (b) [5 marks] Prove that the bound in (a) is sharp, by constructing, for each integer $k \geq 1$, an odd word of length $2^k - 1$ over an alphabet of size k . (This direction is easier, and induction is one good way to prove it.)