

University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2020
Problem Set 2

Distributed Friday, January 17 2020.

Due Friday, January 24 2020 by 5 PM. Hand in to LEARN.

All answers should be accompanied by proofs. In all problems the underlying alphabet Σ is assumed to be finite.

Please use a document preparation system like LaTeX or Word for your solutions. Do not handwrite solutions! For diagrams only, feel free to draw them by hand if you like, and scan them.

1. [10 marks] Find a finite binary word x that contains, as subwords, fewer than $|x|$ distinct nonempty palindromes.
2. [10 marks] Prove the following improvement on Theorem 2.3.6 in the course text. You can use the same idea as in the proof of that theorem.

Let x and y be nonempty words.

Show that $x^\alpha = y^\beta$ for some fractional exponents $\alpha, \beta > 1$ satisfying $\alpha + \beta \leq \alpha\beta$ iff $xy = yx$.

3. [10 marks] An infinite word $\mathbf{x} = a_0a_1a_2\cdots$ is said to be *recurrent* if every subword that occurs in \mathbf{x} occurs infinitely often in \mathbf{x} .
 - (a) [5 marks] Show that an infinite word \mathbf{x} is recurrent iff every subword that occurs in \mathbf{x} , occurs at least twice.
 - (b) [5 marks] Recall that a word \mathbf{w} is “reversal-closed” if it has the following property: x a finite subword of \mathbf{w} implies that x^R is a subword of \mathbf{w} . Show that if an infinite word is reversal closed, then it is recurrent.