University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2020
Problem Set 3

Distributed Friday, January 24 2020.

Due Friday, January 31 2020 by 5 PM. Hand in to LEARN.

All answers should be accompanied by proofs. In all problems the underlying alphabet Σ is assumed to be finite.

Please use a document preparation system like LaTeX or Word for your solutions. Do not handwrite solutions! For diagrams only, feel free to draw them by hand if you like, and scan them.

1. [10 marks] If quotient were a true inverse to concatenation, we might expect it to obey the identities

$$L_1/(L_2L_3) = (L_1/L_2)/L_3$$
$$L_1/(L_2L_3) = (L_1/L_3)/L_2$$

for all languages L_1, L_2, L_3.

For each of these two identities, either prove it holds, or give a counterexample.

2. [10 marks] (A variation on the quotient operation on languages.) Let $L_1, L_2 \subseteq \Sigma^*$, and define $\mathrm{iq}(L_1, L_2)$, the infix quotient of L_1 and L_2, to be the language

$$\{y \in \Sigma^* \ : \ \text{there exist } x, z \in L_2 \text{ such that } xyz \in L_1\}.$$

Prove that if L_1 is regular then $\mathrm{iq}(L_1, L_2)$ is regular.

3. [10 marks] Suppose L_1 and L_2 are finite languages containing m and n strings, respectively, for $m, n \geq 1$.

   (a) Show that the language L_1L_2 has at most mn strings and at least $m + n - 1$ strings.

   (b) Show, by a set of examples, that for each pair of integers $m, n \geq 1$ there exist languages L_1 and L_2, containing exactly m and n strings, respectively, such that L_1L_2 has exactly $m + n - 1$ strings.