

**University of Waterloo**  
**CS 462 — Formal Languages and Parsing**  
**Winter 2020**  
**Problem Set 4**

*Distributed Friday, January 31 2020.*

*Due Friday, February 7 2020 by 5 PM. Hand in to LEARN.*

Please use a document preparation system like LaTeX or Word for your solutions. Do not handwrite solutions! For diagrams only, feel free to draw them by hand if you like, and scan them.

1. [10 marks]

- (a) [4 marks] Construct a finite-state transducer  $T$  that multiplies by 3 in base 2. More precisely,  $T$  should take the canonical representation of  $n$  in base 2, *starting with the least significant digit*, and produce the canonical representation of  $3n$ , again starting with the least significant digit. You can assume the input has no trailing zeros and you must ensure that the output also has no trailing zeros. For example, if the input is 1011, then the output should be 111001. On input  $\epsilon$  the output should be  $\epsilon$ .

Hint: use nondeterminism to handle the correct output on the last few bits. Be sure to label which states are final in  $T$ .

- (b) [4 marks] Do the same thing, but where the input is given *starting with the most significant digit*. Hint: do (a) first, then reverse the direction of the transitions.
- (c) [2 marks] Explain why it is impossible for a *deterministic* transducer to solve the problem in part (b). Hint: what if the input is  $\lfloor 2^n/3 \rfloor$  versus  $\lceil 2^n/3 \rceil$ ?

In all three parts, please briefly justify correctness, but a complete formal proof is not needed.

2. [10 marks] Let  $s_k(n)$  denote the sum of the digits of  $n$ , when expressed in base  $k$ . Thus, for example,  $s_2(5) = 2$ .

Using finite automata, prove that  $s_3(13n) \neq 4$  for all  $n \geq 0$ .

Suggested outline: first construct a DFA  $M_1$  recognizing the base-3 representations of multiples of 13. Next, construct another DFA  $M_2$  recognizing those  $x \in \{0, 1, 2\}^*$  such that the sum of the elements of  $x$  is equal to 4. Next, create  $M_3$  recognizing the intersection of  $L(M_1)$  and  $L(M_2)$ . Finally, observe that in  $M_3$  there is no path from the initial state to a final state.

You can, if you wish, use any software for manipulating automata that you find online. One possibility is **Grail**, available at <http://grail.smcs.upei.ca>. Provide complete details of your computation, including any code that you wrote. A complete formal proof is not needed.

(There are other ways to solve this problem, but I want you to use automata.)

3. [10 marks] Consider the following transformation on languages:

$$\mathbf{sqrt}(L) = \{x \in \Sigma^* : \text{there exists } y \in \Sigma^* \text{ such that } |xy| = |x|^2 \text{ and } xy \in L\}.$$

Show that if  $L$  is regular, then so is  $\mathbf{sqrt}(L)$ . Hint: use the boolean matrix approach and follow the general idea in Proposition 3.8.8.