University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2018
Problem Set 4

Distributed Monday, January 29 2018.

Due Monday, February 5 2018 by 5 PM. Hand in to LEARN.

All answers should be accompanied by proofs. In all problems the underlying alphabet Σ is assumed to be finite.

1. [10 marks]

   (a) [5 marks] Draw a finite-state transducer that, given a word $w \in \{\mathtt{a}, \mathtt{b}\}^*$, replaces each copy of `ab` with `acb`. For example, if the input is `baabab`, the output should be `baacbacb`.

   (b) [5 marks] Using Nivat's theorem, give an expression involving L, some morphisms and inverse morphisms, intersection, and possibly other regular expression symbols, that carries out the action of the transducer in (a) on the language L.

2. [10 marks] (A variation on quotient.) Define $\mathrm{iq}(L_1, L_2)$, the infix quotient of L_1 and L_2, to be the language

$$\{y \ : \ \text{there exist } x, z \in L_2 \text{ such that } xyz \in L_1\}.$$

   Prove that if L_1 is regular then $\mathrm{iq}(L_1, L_2)$ is regular.

3. [10 marks] In this problem you will study pairs of words like `reward` and `drawer`: each is the reverse of the other, but they are not palindromes.

   (a) Show that there is a constant c and an infinite class of DFA's M, each with input alphabet $\{a, b\}$, for which there do exist $u, v \in L(M)$ with $u \neq v$ and $u = v^R$, but the *shortest* such strings in $L(M)$ have length $\geq cn^2$, where n is the number of states of M.

   (b) Show that if an n-state DFA M accepts two strings u, v such that such that $u \neq v$ and $u = v^R$, then it accepts two such strings of length at most $O(n^4)$.

   This is a bit hard! Hint: one possible approach starts as follows: using M, construct an NFA-ϵ M' that accepts, *in parallel*, the *first halves* of all pairs $u, v \in L(M)$ satisfying $u \neq v$ and $u = v^R$.

   There may be other ways to solve it.

4. [10 marks extra credit] I don't currently see how to decrease the $O(n^4)$ upper bound in (b) or increase the $\Omega(n^2)$ lower bound in (a) to make them closer together. Can you?