University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2018
Problem Set 6

Distributed Monday, February 12 2018.

Due Monday, February 26 2018 by 5 PM. Hand in to LEARN. Note: because of the winter break, you have two weeks for this one. But don't put off working on it!

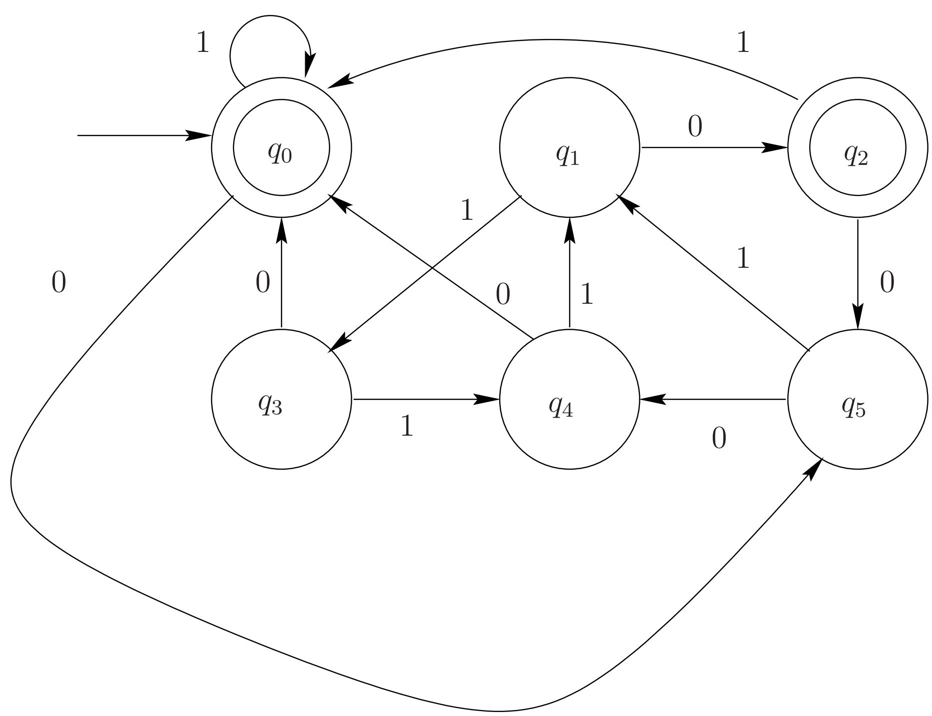
All answers should be accompanied by proofs. In all problems the underlying alphabet Σ is assumed to be finite.

1. [10 marks] A state q of a DFA $M = (Q, \Sigma, \delta, q_0, F)$ is sometimes called "dead" if $\delta(q, x) \notin F$ for all $x \in \Sigma^*$.

   (a) [5 marks] Prove that a minimal DFA has at most one dead state.

   (b) [5 marks] Give an example of a minimal DFA with input alphabet $\Sigma = \{0, 1\}$, but without any dead states.

2. [10 marks] Using any method discussed in class, find the minimal DFA equivalent to the one whose transition diagram is given below. Show all steps.



3. [10 marks] A string of length n over the alphabet $\{0, 1, 2, \ldots, n-1\}$ is called *Bell* if it has the property that the first letter is 0, and each succeeding letter is at most 1 more than the maximum of all preceding letters. For example, 00102 is Bell, but 00201 is not (since the symbol 2 violates the condition).

   (By the way, the number of such length-n strings is counted by the Bell numbers — see the Wikipedia article `http://en.wikipedia.org/wiki/Bell_number` — hence the name.)

   Let B_n be the language of all Bell strings of length n.

   Here is B_3: $\{000, 001, 010, 011, 012\}$.

   (a) [5 marks] Show that B_n can be accepted by a DFA with $O(n^2)$ states.

   (b) [5 marks] Using the Myhill-Nerode theorem, show that any DFA accepting B_n has at least $\Omega(n^2)$ states.

4. [10 marks — extra credit only] Find good nontrivial upper and lower bounds on the size of the shortest *regular expression* for the language B_n of problem 3 above. I know there is an expression of size $O(2^n)$, but can you do significantly better? (I get to decide what constitutes a "good nontrivial" solution.)