1. [10 marks] Using any minimization method we discussed in class, find a minimal DFA equivalent to the DFA depicted below. Show your work.

2. [10 marks] In one edition of Peter Linz’s book, *Introduction to Formal Languages*, he gives the following exercise: find a DFA for the language

$$L = \{w \in \{a, b\}^* : |w|_a - |w|_b \mod 3 < 2\}.$$ 

But $L$ is actually not regular! Prove that it isn’t using the Myhill-Nerode theorem.

Note: $a \mod b$ is the least **non-negative** remainder in the division of $a$ by $b$, so that, for example, $(-1) \mod 3 = 2$. Also, the absolute value signs mean arithmetical absolute value on the outside, but on the inside $|w|_a$ means the number of occurrences of $a$ in $w$. 
3. [10 marks] For each integer $n \geq 1$ compute, with proof, the deterministic state complexity (= minimum number of states in a DFA accepting it) of the language

$$L_n = \{ x \in \{0, 1\}^* : [x]_2 \equiv 0 \pmod{n} \},$$

where by $[x]_2$ we mean the integer represented in base 2 by the string $x$. Note: $x$ is allowed to have leading 0’s.