All answers should be accompanied by proofs. In all problems the underlying alphabet \( \Sigma \) is assumed to be finite.

1. [10 marks] Using any method, find a minimal DFA equivalent to the DFA depicted below. Show your work.

2. [10 marks] Prove or disprove: any DFA accepting a regular language \( L \) must have at least as many final states as the number of final states in the minimal DFA for \( L \).

3. [10 marks] For each integer \( n \geq 1 \) compute the deterministic state complexity (size of the minimal DFA) for the language \( L_n = \{ x \in \{0, 1\}^* : [x]_2 \equiv 0 \pmod{n} \} \), where by \( [x]_2 \) we mean the integer represented in base 2 by the string \( x \). Note: \( x \) is allowed to have leading 0’s, and \( [\epsilon]_2 = 0 \).