University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2020
Problem Set 7

Distributed Friday, February 28 2020. Due Friday March 6 2020 at 5 PM. Submit via LEARN.

1. [10 marks] Consider the positive squares, represented in base 6. Here are the first few elements of this language:

$$L = \{\mathtt{1}, \mathtt{4}, \mathtt{13}, \mathtt{24}, \mathtt{41}, \mathtt{100}, \mathtt{121}, \mathtt{144}, \mathtt{213}, \mathtt{244}, \ldots\}.$$

   What are the minimal elements of L under the subsequence order? Prove your answer.

2. [10 marks] Call a morphism $h : \Sigma^* \to \Sigma^*$ non-erasing if $h(a) \neq \epsilon$ for all $a \in \Sigma$.

   Let the domain be $S = \{\epsilon\} \cup 0\{0,1\}^*$. These are the binary strings that do not begin with a 1.

   Consider the following order defined on S: for strings $x, y \in S$ we say $x \leq y$ if there is some non-erasing morphism $h : \Sigma^* \to \Sigma^*$ such $h(x) = y$.

   Examples: $010 \leq 000$ because we can take $h(0) = h(1) = 0$.

   $010 \leq 1001$ because we can take $h(0) = 1$, $h(1) = 00$.

   But 0110 is not $\leq$ 0101, because whatever we decide to map 1 to must be doubled in its image under h. So it must be that $h(1) = 01$, the only doubled string in 0101. But since $h(0)$ is nonempty, this says $h(0110)$ must be of length at least 6, contradiction.

   (a) Show that $\leq$ is a partial order on S.

   (b) Recall that an *antichain* is a set of pairwise incomparable elements. Show that $\leq$ has infinite antichains.

3. [10 marks] If $L \subseteq \Sigma^*$ is a CFL, need

$$\{x \in \Sigma^* \ : \ x^* \subseteq L\}$$

   be a CFL? Prove or give a counterexample.