Lecture 15. Deterministic Context-Free Languages

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University of Waterloo
Meng He
Definitions

- **Deterministic Pushdown Automaton (DPDA):** A PDA \( M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F) \) is deterministic if:
  - \( |\delta(q, a, A)| \leq 1 \) for all \( q \in Q, a \in \Sigma \cup \{\varepsilon\}, \) and \( A \in \Gamma \)
  - For all \( q \in Q \) and \( A \in \Gamma \), if \( \delta(q, \varepsilon, A) \neq \emptyset \), then \( \delta(q, a, A) = \emptyset \) for all \( a \in \Sigma \).

- Why didn’t we allow \( \varepsilon \)-moves for DFA’s?

- A language accepted by a DPDA by final state is a **DCFL**
  - A DCFL is also a CFL, but the converse is not true
The class of DCFL is closed under complement

The reasons why we cannot simply use the idea of proving the closure under complement for regular languages:

1. A DPDA $M$ may enter a state in which it never consumes additional input symbols
   1) $M$ has no defined move
   2) The stack has been emptied
   3) $M$ enters an infinite loop on $\varepsilon$-transitions

2. After reading an input string, a DPDA $M$ may enter a sequence of states on $\varepsilon$-transitions, among which there is at least one final state and at least one non-final state.

Forcing DPDA’s to scan their input. If $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ is a DPDA, then there exists a DPDA $M' = (Q', \Sigma, \Gamma', \delta', q_0', X_0, F')$ such that $L(M) = L(M')$ and $M'$ always scans its entire input.
More on Closure Properties

- Every DCFL is accepted by some DPDA that, in an accepting state, may make no move on \( \varepsilon \)-input.
- Application. \( L = \{w \in \{a, b\}^*: w \neq xx \text{ for all } x \in \{a, b\}^*\} \) is not a DCFL.
- The class of DCFL is not closed under union or intersection (see Assignment 4).
- The class of DCFL is closed under intersection with regular languages.
Myhill-Nerode Equivalence Relation and DCFL

- Let $L$ be a language such that each Myhill-Nerode equivalence class is of finite cardinality. Then $L$ is not a DCFL.

- Application. The language $PAL = \{x \in \{a,b\}^*: x = x^R\}$ is not a DCFL.