

Problem-Solving Session 8, Question 4

A.S.R.

March 21, 2017

The claim that $\psi(\overline{R}) = \mathbb{N}^k - \psi(R)$ is false. We prove this by a simple contradiction. Assume the statement is true for all regular languages R .

Consider $R = \{01\}$ on the binary alphabet. Clearly, $\psi(R) = \{(1,1)\}$. From the assumption, $\psi(\overline{R}) = \mathbb{N}^2 - (1,1)$. We thus conclude that no word whose Parikh map is $(1,1)$ can be in \overline{R} .

However, the language $\overline{R} = \Sigma^* - \{0,1\}$ contains 10 , whose Parikh map is $(1,1)$. This contradiction proves that the assumption is false. That is the mistake in the proof.