The claim that $\psi(R) = N^k - \psi(R)$ is false. We prove this by a simple contradiction. Assume the statement is true for all regular languages $R$.

Consider $R = \{01\}$ on the binary alphabet. Clearly, $\psi(R) = \{(1, 1)\}$. From the assumption, $\psi(\overline{R}) = N^2 - (1, 1)$. We thus conclude that no word whose Parikh map is $(1, 1)$ can be in $\overline{R}$.

However, the language $\overline{R} = \Sigma^* - \{0, 1\}$ contains 10, whose Parikh map is $(1, 1)$. This contradiction proves that the assumption is false. That is the mistake in the proof.