ASSIGNMENT 1

DO NOT COPY. ACKNOWLEDGE YOUR SOURCES.

1. [5 marks] In class we saw a polynomial-time 2-approximation algorithm for the metric Travelling Salesman Problem (TSP). Prove that if there is a polynomial-time $k$-approximation algorithm for general TSP then P=NP. In more detail: Assume you have an algorithm with a polynomial running time that takes an input for TSP and returns a tour whose length is guaranteed to be at most $k$ times the optimum. Prove that you can then solve a known NP-complete problem—specifically the Hamiltonian cycle problem—in polynomial time.

2. [5 marks] Suppose we wish to maintain an array of elements subject to the following two operations:

- $add(e)$ – add element $e$ to the next available space in the array. The cost is 1 time unit.
- $empty$ – empty the array by removing all its elements. The cost is $k + 1$ time units if the array has $k$ elements in it.

The goal of this question is to use the potential method to prove that the amortized cost of each operation is $O(1)$ in any sequence of operations starting with an empty array.

Define the charge of the $add$ operation to be 2. Define the charge of the $empty$ operation to be 1. Recall that the definition of the potential after the $i^{th}$ operation is $\Phi_i = \Phi_{i-1} + charge(i) - cost(i)$, with the initial potential $\Phi_0 = 0$.

(a) [3 marks] What is the potential when the array has $k$ elements?

(b) [1 mark] Prove that the final potential is $\geq$ initial potential.

(c) [1 mark] Conclude that the amortized cost of each operation is $O(1)$. (You can quote the relevant theorem from class, you do not need to reprove it.)

Note that the point of this exercise is to use the potential method. Do not answer the question using a different method.