CS 466/666 Fall 2018 due Tuesday October 2, 5 PM
A. Lubiw

ASSIGNMENT 2

DO NOT COPY. ACKNOWLEDGE YOUR SOURCES.

1. [10 marks] Suppose you have a splay tree with keys $1, \ldots, n$, and you access the keys sequentially, i.e. you search for $1, 2, \ldots, n$, in that order, performing a splay on each one.

   (i) [2 marks] What is the structure of the final splay tree? Justify your answer.

   One remarkable property of splay trees is that the total work for this access sequence is $O(n)$. This is called the Sequential Access Property. It is hard to prove.

   (ii) [2 marks] Prove the following special case: Suppose that the initial splay tree is a path of right children. Prove that the the total work for the access sequence $1, 2, \ldots, n$ is $O(n)$.

   The Dynamic Optimality Conjecture says that, for any access sequence, splay trees have a cost as good (within a constant) as the optimum cost of searching with any rotations interspersed (each rotation costs 1). The Dynamic Optimality Conjecture implies the Sequential Access Property, as you will now show:

   (iii) [5 marks] Show how to use single rotations to turn any binary search tree into a path of right children in $O(n)$ time.

   (iv) [1 mark] Prove that the Dynamic Optimality Conjecture implies the Sequential Access Property.

2. [7 marks] A grad student named M.A. Zing has discovered the remarkable property that Binomial Heaps have $O(1)$ amortized cost for all three operations, merge, insert, and delete-min. They defined the potential of a binomial heap to be the number of trees plus the rank of the largest tree. So, for example, a binomial heap on 19 elements (with binary representation 10011) has potential $3 + 4 = 7$. Using this potential function, M.A. Zing proved that the merge operation has amortized cost $O(1)$. (This part is true, and I would ask you to confirm it but the assignment is long enough already.) Next, M.A. Zing claimed that clearly, since insert and delete-min are implemented using one merge each, they have amortized cost $O(1)$ as well.

   (a) [3 marks] Show that this is impossible using the fact that sorting a list of n elements requires $\Omega(n \log n)$ time.

   (b) [4 marks] Explain precisely what is wrong with M.A. Zing's argument.