CS 466/666 Fall 2018 — due Thursday November 29, 5 PM

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ASSIGNMENT 8

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1. [10 marks] Consider the following rather stupid caching algorithm. Suppose a page is requested and it is not in the cache. If there is no empty slot in the cache, then evict all the pages in the cache. Put the requested page into the cache.

   Prove that this algorithm has competitive ratio k. Here k is the size of the cache, and the cost model is to charge 1 every time a page is evicted from the cache.

2. [10 marks] Consider an auction where the bids come one-by-one (not in sorted order). Each bid is an integer in the range $[1, \ldots, B]$. The algorithm must decide after each bid is made whether to accept it or not. As soon as the algorithm accepts a bid, the auction stops. An optimum offline algorithm will pick the maximum bid. Analyze the competitive ratio of the following two online algorithms. Note that this is a maximization problem so we want a lower bound on ALG/OPT.

   (a) [5 marks] **A deterministic algorithm when the number of bids is known in advance.**

   Define a *threshold value* $T = \sqrt{B}$. The algorithm accepts the first bid $\geq T$ if there is one, otherwise it accepts the last bid. Prove that the competitive ratio is at least $\frac{1}{\sqrt{B}}$. To do this, let M be the maximum bid, i.e., OPT$= M$, and consider two cases, depending whether $M \geq T$ or not.

   (b) [5 marks] **A randomized algorithm that works even if the number of bids is not known in advance.**

   The algorithm picks i at random from $[1, 2, \ldots, \log B]$, and sets the threshold T to be 2^{i-1}. Then the algorithm accepts the first bid $\geq T$ if there is one. In the worst case, there is no bid $\geq T$ and the algorithm gets 0.

   Prove that the expected competitive ratio is at least $\frac{1}{2 \log B}$. Hint: Let M be the maximum bid, and consider k such that $2^k \leq M < 2^{k+1}$.