Hedging (Dynamic Trading) Analysis

In addition to pricing: MC simulation allows us to analyze complex problem, e.g., dynamic trading performance

Consider option hedging:

• Hedging is a dynamic trading process and trading can only be implemented at discrete times
• Hedging is crucial for financial institutions. Analyzing performance of hedging/dynamic trading process is very important exercise.

Goal of analysis:

• How good is your hedging/trading strategy?
• How risky is it? (how much risk remains for hedging)
• How do we measure the risk?

It is important to conduct

• hedging analysis
• back testing

We conduct model based hedging analysis, assuming model is correct, selling an option, dynamically trading underlying, and cash account financing.

This corresponds to the dynamic trading strategy \{-V_t, \delta_t S_t, B_t\}
where the bond account always ensures balancing of the account (self-financed)

Assume

\[ \frac{dS}{S} = \mu dt + \sigma dZ_t \]

We generate MC scenario paths based on the above model and compute P&L of the strategy along each path for the dynamic trading strategy considered.

**Note.** Note risk neutral model

Assume that trading time are \( t_n = n\Delta t, \Delta t = \frac{T}{N}, \) where

\[ 0 = t_0 < t_1 < \cdots < t_N = T \]

Assume that \( \delta_n \) units of \( S \) are held in \([t_n, t_{n+1}]\).

**Initially** \( n = 0 \), option: \( -V_0 = -V(S_0,0) \), underlying \( \delta_0 \), balancing with cash account by setting

\[ B_0 = V_0 - \delta_0 S_0 \]

Portfolio \( \Pi = -V + \delta S + B \) has initial value \( \Pi_0 = 0 \)

At \( t_n \), \( \Pi_n = -V(S_n,t_n) + \delta_n S_n + B_n \)

Consider \([t_n, t_{n+1}]\). At \( t_{n+1} \), rebalancing position in share and updating cash account so that \( \Pi_{t_{n+1}}^- = \Pi_{t_{n+1}}^+ \)

\[ -V(S_{n+1},t_{n+1}) + \delta_n S_{n+1} + B_n e^{r\Delta t} = -V(S_{n+1},t_{n+1}) + \delta_{n+1} S_{n+1} + B_{n+1} \]
\[ B_{n+1} = B_n e^{r \Delta t} + (\delta_n - \delta_{n+1}) S_{n+1} \]

If \( \delta_{n+1} > \delta_n \), buy additional \( \delta_{n+1} - \delta_n \) units.

If \( \delta_{n+1} < \delta_n \), sell additional \( \delta_{n+1} - \delta_n \) units.

At expiry \( t_N = T \), liquid the portfolio formed at \( t_{N-1} \), which has value

\[ \Pi_N = -V(S_N, t_N) + \delta_{N-1} S_N + B_{N-1} e^{r \Delta t} \]

Note.

- \( \Pi_N \) is random. If \( \Pi_N \equiv 0 \), \( \Rightarrow \) perfect hedge
- \( \Pi_N > 0 \), a profit scenario for the writer
- \( \Pi_N < 0 \), a loss scenario.
- \( \Pi_N \) is a measure of hedging error. For hedging, we often consider discounted relative P&L

\[ P&L = \frac{e^{-rT} \Pi_N}{V(S_0, 0)} \]

We then examine distribution properties of P&L

**How to Compute Delta for Hedging?**

**Delta Hedging:** at \( t \), \( \delta_t = \frac{\partial V}{\partial S}(S(t), t) \)

In general, we only have approximations to delta.

If a binomial lattice is used, solving replication equation yields
\[ \delta_j^n = \frac{V_{j+1}^{n+1} - V_j^{n+1}}{(u - d)S_j} \]