CS 497: Electronic Market Design

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Auctions

- Methods for allocating goods, tasks, resources, ...
- Participants
  - auctioneer
  - bidders
- Enforced agreement between auctioneer and the winning bidder(s)
- Easily implementable (e.g. over the Internet)
- Conventions
  - Auction: one seller and multiple buyers
  - Reverse auction: one buyer and multiple sellers
Auction Settings

- **Private value**: the value of the good depends only on the agent’s own preferences
  - e.g. a cake that is not resold or showed off
- **Common value**: an agent’s value of an item is determined entirely by others’ values (valuation of the item is identical for all agents)
  - e.g. treasury bills
- **Correlated value (interdependent value)**: agent’s value for an item depends partly on its own preferences and partly on others’ value for it
  - e.g. auctioning a transportation task when bidders can handle it or reauction it to others
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Four Common Auctions

- English auction
- First-price, sealed-bid auction
- Dutch auction
- Vickrey auction
English auction
aka first-price open-cry auction

- **Protocol:** Each bidder is free to raise their bid. When no bidder is willing to raise, the auction ends and the highest bidder wins. Highest bidder pays its last bid.

- **Strategy:** Series of bids as a function of agent’s private value, prior estimates of others’ valuations, and past bids

- **Best strategy:**

- **Variations:**
  - Auctioneer controls the rate of increase
  - Open-exit: Bidders have to openly declare exit with no re-entering possibilities
First-price sealed-bid auction

- **Protocol**: Each bidder submits one bid without knowing others’ bids. The highest bidder wins the item at the price of its bid
- **Strategy**: Bid as a function of agent’s private value and its prior estimates of others’ valuations
- **Best strategy**: 
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- **Best strategy:**
Example

Assume there are 2 agents (1 and 2) with values $v_1, v_2$ drawn uniformly from $[0, 1]$. Utility of agent $i$ if it bids $b_i$ and wins is $u_i = v_i - b_i$.

Assume that agent 2’s bidding strategy is $b_2(v_2) = v_2/2$. How should 1 bid? (i.e. what is $b(v_1) = z$?).

\[ U_1 = \int_{z=0}^{2z} (v_1 - z)dz = (v_1 - z)2z = 2zv_1 - 2z^2 \]

Note: given $z = b_2(v_2) = v_2/2$, 1 only wins if $v_2 < 2z$

Therefore,

\[ \text{arg max}_z[2zv_1 - 2z^2] = v_1/2 \]

Similar argument for agent 2, assuming $b_1(v_1) = v_1/2$. 
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Example

Assume that there are 2 risk-neutral bidders, 1 and 2.

- Agent 1 knows that 2’s value is 0 or 100 with equal probability
- 1’s value of 400 is common knowledge

What is a Nash equilibrium?
Dutch (Aalsmeer) flower auction
Dutch auction
Descending auction

- **Protocol:** Auctioneer continuously lowers the price until a bidder takes the item at the current price
- **Strategy:** Bid as a function of agent’s private value and prior estimates of others’ valuations
- **Best strategy:**
  - Dutch flower market, Ontario tobacco auctions, Filene’s basement,...
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Vickrey Auction
aka Second price, sealed bid auction

Protocol: Each bidder submits one bid without knowing the others’ bids. The highest bidder wins and pays an amount equal to the second highest bid.

Strategy: Bid as a function of agent’s private value and its prior estimates of others’ valuations.

Best strategy:
- Widely advocated for computational multiagent systems
- Old (Vickrey 1961) but not widely used by humans
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The Vickrey auction is a special case of the Clarke Tax.

- **Who pays?**
  - The bidder who takes the item away from the others (making the others worse off)
  - Others pay nothing

- **How much does the winner pay?**
  - The declared value that the good would have had for the others had the winner stayed home (second highest bid)
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Results for Private Value Auctions

- Dutch and first-price sealed-bid auctions are strategically equivalent
- For risk neutral agents, Vickrey and English auctions are strategically equivalent
  - Dominant strategies
- All four auctions allocate item efficiently
  - Assuming no reservation price for the auctioneer
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Revenue

Theorem (Revenue Equivalence)

Suppose that
- values are independently and identically distributed and
- all bidders are risk neutral.

Then any symmetric and increasing equilibrium of any standard auction, such that the expected payment of a bidder with value zero is zero, yields the same expected revenue.

Revenue equivalence fails to hold if agents are not risk neutral.
- Risk averse bidders: Dutch, first-price $\geq$ Vickrey, English
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Sniping

Figure 1a–Cumulative distributions over time of bidders’ last bids
Figure 1b–Cumulative distributions over time of auctions’ last bids
Sponsored Search
Advertisers are ranked and assigned slots based on the ranking.

If an ad is clicked on, only then does the advertiser pay.
Sponsored Search

Rank-by-relevance

- Assign slots in order of ($bid$)($quality$ score)

<table>
<thead>
<tr>
<th>Bidder</th>
<th>Bid</th>
<th>Quality Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.50</td>
<td>0.5</td>
</tr>
<tr>
<td>B</td>
<td>1.00</td>
<td>0.9</td>
</tr>
<tr>
<td>C</td>
<td>0.75</td>
<td>1.5</td>
</tr>
</tbody>
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<th>Ranking</th>
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<tr>
<td>A (0.75)</td>
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</tbody>
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A bidder only pays when its ad is clicked on
How much does it pay?
   The lowest price it *could* have bid and still maintained its rank
Sponsored Search

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C will pay $p = 0.9/1.5 = 0.6$
B will pay $p = 0.75/0.9 = 0.8$
A will pay ?
There are many questions about sponsored search

- Is the current way (Generalized Second Price Auction) the best way?
- Revenue?
- Pay-per-what?
- Fraud/vindicitive behavior?
- Budgets?
- Should bidders understand how the auction works?
- ...
Selling Multiple Items

So far we have only talked about auctioning a single item. What if we want to sell multiple items?
Multiple Items

- Parallel Auctions
- Sequential Auctions

In both these approaches you have the *exposure problem*. 
Multiple Items

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- Sequential Auctions

In both these approaches you have the *exposure problem*. 
Combinatorial Auctions

Allow bidders to submit bids on *bundles* of items.

\[<(\text{coffee, donut, } $5.00)\text{XOR (cake, tea, } $4.50)\text{XOR ...}>\]

- Allocation \( x^* = \arg\max_x \sum_i^n v_i(x) \) where \( v_i \) is the bid of agent \( i \)
- Payment \( p_i = \sum_{j \neq i} v_j(x') - \sum_{j \neq i} v_j(x^*) \) where \( x' \) is the allocation if bidder \( i \) had not participated.
- Efficient and truthful!
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To run a Combinatorial Auction we must solve

\[ x^* = \underset{x}{\arg \max} \sum_{i}^{n} v_i(x) \]

- Weighted Set-Packing Problem
- No PTAS
Winner Determination Problem

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- Special structure in the bids
  - Limiting choices for the bidders
- Approximations and heuristics for the WDP
  - Can interfere with the incentive properties of the VCG mechanism
- Throw lots of computing power at the problem

Other issues include
- Communication and preference elicitation
- Design of iterative auctions
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Other research problems

- Computational Limitations and Bidding Behaviour
- Trading Agent Design (Trading Agent Competition)
- Market Design (CATS)
- Trust and Reputation in Online Markets
- Incentive-based computing
- ...

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